A Study of Subharmonic Injection Locking for Local Oscillators

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***Abstract*—The analysis of a subharmonic injection locked local oscillator introduced here is based on a general nonlinear input-output model for the subharmonic synchronized oscillator. The results show the nth-order subharmonic injection locking oscillator is locked primarily by the nth harmonic output of injected signal that is generated by the current-voltage nonlinearity of the active device. The measurement of subharmonic injection locking range, at factors of 1/2, 1/3, and 1/4, of a MESFET DRO verified these results.**

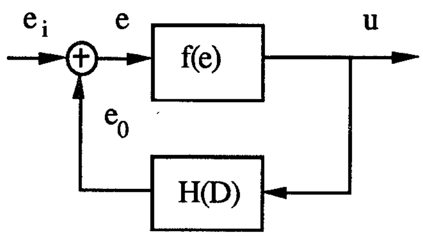

Fig. 1. Conceptual diagram model of subharmonically synchronized oscillator.

I. Introduction

The directly modulated fiber-optic link is limited in bandwidth, and subharmonic injection locking has been demonstrated as a practical technique for optical synchronization of remotely located oscillators at millimeter-wave frequencies [1], [2]. Therefore, the study of the nonlinear characteristics of solid state devices and the effect of oscillator circuit topology on the subharmonic injection locking figures of merit is critical in establishing an efficient synchronization method.

The nonlinear model for injection locking oscillator based on the Van der Pol's representation is well known [3]. However, this representation is not easily implemented for microwave oscillators because of the one-port topology and the small perturbation signal assumption. To solve this problem, Daryoush *et al.* [2], [4] used a general nonlinear model to analyze the subharmonic injection locking range of a microwave oscillator in terms of the nonlinear current-voltage relationship of an active device. This model differs from the Van der Pol mathematical representation in that Daryoush uses a two-port model that is more practical and has the capability to handle large injection signal levels.

This letter expands on this approach in that we have considered the contribution from higher order nonlinearities and derived a general expression to predict the subharmonic injection locking range. We prove analytically that the phenomena for the nth-order subharmonic injection locking process are explained as follows: 1) the nonlinear device generates the nth harmonic response of injected signal; 2) this signal locks the free-running oscillator similar to the fundamental injection locking. By taking advantage of this explanation, we can design local oscillators with good injection locking range by optimizing the multiplication factor of the nonlinear active device.

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II. Subharmonic Injection Locking—Analysis

The nonlinear circuit of oscillator, depicted in Fig. 1, is modeled as a combination of a pure nonlinear network *f*(*e*) and a pure linear feedback network *H*(*D*) [2]. A linear single tuned network *H*(*D*) can be expressed as approximately

$$H(D) = \frac{H_0}{1 + j2Q\frac{\Delta\omega}{\omega_0}}, \tag{1}$$

where Q is the quality factor, and $\Delta\omega$ is the frequency deviation form the resonating frequency ω_0 of feedback circuit. Clearly, it is desirable that the output of this network, e_0, be a sinusoidal signal. When the signal e_i is injected, the input signal e for nonlinear network is

$$e = e_0 + e_i = \frac{E}{2}(e^{j\omega t} + e-j\omega t) + \frac{\dot{E}_i}{2}e^{j\frac{\omega}{n}t}, \tag{2}$$

where $\omega = n\omega_{\text{inj}}$ is the synchronized frequency after injection locking, and ω_{inj} is the injection frequency, Phasor $\dot{E}_i$, the injected signal, is represented by the amplitude $\dot{E}_i$ and phase θ, n is an integer for the subharmonic factor, E is the oscillation signal's amplitude at input port. The output of the oscillator can be expanded in a Fourier Series:

$$u = f(e) = \sum_{m=-\infty}^{\infty} \dot{U}_m e^{jm\frac{\omega}{n}t}. \tag{3}$$

To simplify the analysis, $f(e)$ is expressed approximately by

$$u = f(e) = \sum_{i=1}^{\infty} \alpha_i e^i, \tag{4}$$

where α_i is considered to be real for simplicity. Substituting (4) into (3), for subharmonic {injection at a factor of n, we

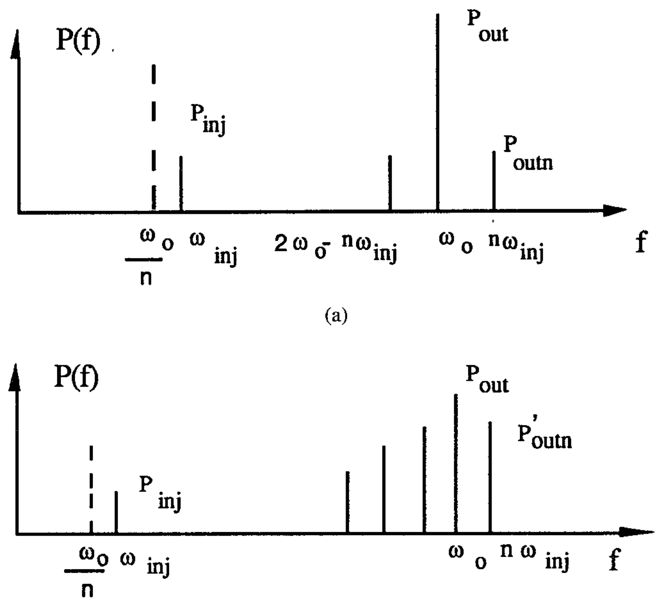

Fig. 2. Oscillator output spectra. (a) Spectra that is used to measure nonlinearity using nth harmonic response. (b) Spectra of unlocked oscillator using subharmonic injection.

have an output signal $\dot{U}_n$ at the oscillation frequency $n\omega_{\mathrm{inj}}$:

$$\dot{U}_n = \left(\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{2^{N-1}}\frac{N!}{(j!)^2(k+1)!k!}\alpha_N|E_i|^{2J}E^{2k}\right)E + \left(\sum_{m=0}^{\infty}\sum_{p=0}^{\infty}\frac{1}{2^{M-1}}\frac{M!}{(m!)^2(p+n)!p!}\alpha_M|E_i|^{2P}E^{2m}\right)\dot{E}_i^n + \text{higher order terms}, \quad (5)$$

where $N = 2j + 2k + 1$ and $M = 2m + 2p + n$. The first term is the oscillation signal amplitude U_{out}, the second term represents the response $\dot{U}_{\mathrm{out}n}$ of injected signal E_i when it goes through the nonlinear network together with the oscillation signal E. The (5) can be simplified as follows:

$$\dot{U}_n \approx U_{\mathrm{out}} + \dot{U}_{\mathrm{out}n}. \quad (6)$$

The subharmonic injection locking range can be expressed in terms of Q and ω_0 by using (1) and letting $n\phi = \pm\pi/n$ [2]:

$$\Delta\omega_{\frac{1}{n}} \approx \frac{\omega_0}{2Q}\frac{U_{\mathrm{out}n}}{U_{\mathrm{out}}} = \frac{\omega_0}{2Q}\sqrt{\frac{P_{\mathrm{out}n}}{P_{\mathrm{out}}}}. \quad (7)$$

Clearly, (7) is the same as Alder's expression for a fundamental injection locking range, when the signal $U_{\mathrm{out}n}$ interacts with the free-running oscillator like an injected fundamental locking signal. Therefore, the subharmonic injection locking process here is identical with that already explained in the introduction.

III. Subharmonic Injection Locking—Experiment Verification

For verification of this analysis, a 5-GHz DRO was designed and fabricated. This oscillator consists of a two-stage low-noise MESFET amplifier and a dielectric resonator between

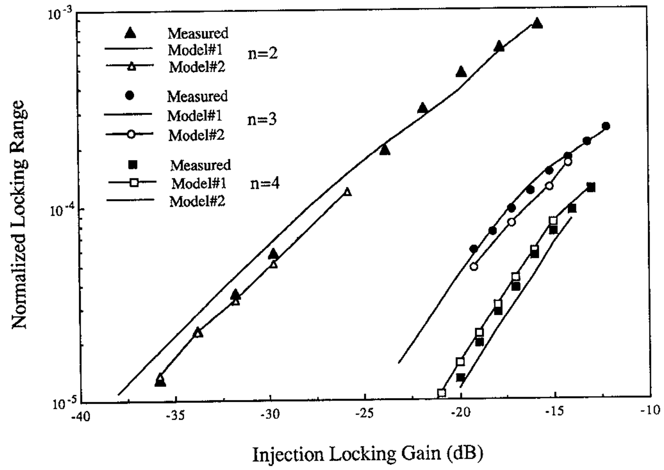

Fig. 3. Measured and calculated injection locking range ($\frac{\Delta f}{f_0}$) at a subharmonic factor n=2, 3, 4, where the calculation for Model #1 represents the calculated results with (7), and the calculation for Model #2 represents the predicted results with the unlocked spectra.

the input and output transmission lines as linear feedback network, layout details of which are discussed in Berceli *et al.* [4]. The output power of the free-running DRO oscillator is 15 dkBm, and the phase noise is about −61 dkBc/Hz, −80 dBc/Hz, and −90 dBc/Hz at 10 KHz, 50 KHz and 100 KHz offset carrier frequency, respectively.

For prediction of the locking range, $P_{\mathrm{out}n}$ was measured according to the following procedure: the slave oscillator was injection locked at the frequency ω_0 by a very strong fundamental signal so that no other signal can significantly influence the oscillation frequency; then an injection signal was injected at frequency ω_{inj} close to the nth subharmonic of ω_0. As shown in Fig. 2(a), the harmonic response of the injected signal P_{out}can be measured. Because the oscillation signal E remains constant in the second term of (5) while the injection signal changes, $P_{\mathrm{out}n}$ can be expressed in terms of the injection power P_{inj} in such a power series as

$$P_{\mathrm{outs}}^{1/2} = P_{\mathrm{inj}}^{n/2}\sum_{m=0}^{\infty}\beta_m P_{\mathrm{inj}}^m, \quad (8)$$

where β_m is directly traceable to α_i by use of (5). By measuring $P_{\mathrm{out}n}$ at a different P_{inj}.

The first few terms of β_m can be fitted. Then the locking range can be predicted in terms of the injected power. Fig. 3 shows the comparison between the measured and predicted locking range at subharmonic factors of 1/2, 1/3, and 1/4, where a good match was obtained between two results.

On the basis of the physical explanation of subharmonic injection locking process, other classical formulas for fundamental injection locking, such as Armand's [5] and Stover's [6] method, can be used to predict the subharmonic injection locking range. By observing the sidebands generated by the unlocked oscillator which is shown in Fig. 2(b), the injection locking ranges were calculated through Armand's and Stover's approaches. Both approaches lead to the same results in our experiment. The calculated locking ranges again agree well with the measured results, as shown in Fig. 3.

IV. Conclusion

Because comparisons in Fig. 3 verify the results in (7), the subharmonic locking range can be easily predicted if the multiplication property of a nonlinear network is known. Also, a subharmonically synchronized local oscillator capable of synchronization to a large subharmonic factor with the optimum locking range can be built, if a nonlinear circuit topology having an optimum multiplication property for a signal at a subharmonic factor is designed. The noise behavior of subharmonically locked LO's is under consideration and will be discussed in a separate paper [7].

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